Chapter 7.3

pothesis Testing for the Mean μ (σ unknown)

Recall from chapter 6: When σ was known, we used a critical value z_c for our confidence interval.
When σ was unknown, we used a critical value t_c for our confidence interval.

Similarly, for chapter 7: When σ known -> Test statistic

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

When σ unknown -> Test statistic

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

7.3

Critical Values in a t-Distribution

In many real-life situations, it is impossible or too expensive to calculate the population standard deviation, so in these cases σ is not known. When either the population has a normal distribution or the sample size is at least 30, you can still test the population mean μ. Remember from Chapter 6, you can use a t-distribution with $n-1$ degrees of freedom.

GUIDELINES

Finding Critical Values in a t-Distribution

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. When the hypothesis test is
   a. *left-tailed,* use the "One Tail, α" column with a negative sign.
   b. *right-tailed,* use the "One Tail, α" column with a positive sign.
   c. *two-tailed,* use the "Two Tails, α" column with a negative and a positive sign.

See the figures below.

If the degrees of freedom you need is not in the table, use the closest number in the table that is **less than** the value you need.

Examples 1-6: Find the critical value(s) and rejection region(s) for the type of t-test with level of significance αand sample size n. State the rejection regions.

1. Left-tailed, $\alpha = 0.05$, $n = 21$

df = 20

$-t_0$

$t_0 = -1.725$

rej reg: $t < -1.725$

2. Right-tailed, $\alpha = 0.01$, $n = 35$ df = 34

t_0

$t_0 = 2.441$

rej reg: $t > 2.441$

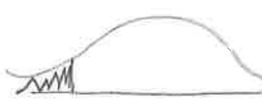
3. Two-tailed, $\alpha = 0.10$, $n = 50$ (df = 49 look at 45)

$-t_0$ t_0

$t_0 = \pm 1.679$

$t < -1.679$, $t > 1.679$

4. Left-tailed, $\alpha = 0.025$, $n = 12$ df = 11



$-t_0$

$t_0 = -2.201$

crit $t < -2.201$

5. Right-tailed, $\alpha = 0.10$, $n = 55$ df = 54 (look up 50)

t_0

$t_0 = 1.299$

rej reg: $t > 1.299$

6. Two-tailed, $\alpha = 0.20$, $n = 47$ df = 46 use 45

$-t_0$ t_0

$t_0 = \pm 1.301$

$t < -1.301$, $t > 1.301$

z - 2 decimal places
t - 3 decimal places

Hypothesis Testing using a Rejection Region:

$\mu \geq 21000$

Example 7: A used car dealer says that the mean price of used cars sold in the last 12 months is at least \$21,000. You suspect this claim is incorrect and find that a random sample of 14 used cars sold in the last 12 months has a mean price of \$19,189 and a standard deviation of \$2950. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed.

a. Identify the claim and state H_0 and H_a.

$H_0: \mu \geq 21000$ (claim)

$H_a: \mu < 21000$

b. Find the critical value(s) and identify the rejection region(s). $df = 13$ $\alpha = .05$

left-tailed

$t_0 = -1.771$

rej. reg: $t < -1.771$

c. Find the standardized test statistic t.

$\mu = 21000$ $\bar{x} = 19189$ $s = 2950$ $n = 14$

$$t = \frac{19189 - 21000}{\frac{2950}{\sqrt{14}}} = -2.297$$

d. Decide whether to reject or fail to reject the null hypothesis

Since t is $< t_0$, it falls in the rej. reg.

reject H_0

e. Interpret the decision in the context of the original claim.

There is enough evidence at the 5% level of significance to reject the dealer's claim that the mean price of used cars sold in the last 12 months is at least \$21,000.

Example 8: An industrial company claims that the mean conductivity of the river is 1890 milligrams per liter. The conductivity of water sample is a measure of the total dissolved solids in the sample. You randomly select 39 water samples and measure the conductivity of each. The sample mean and standard deviation are 2350 milligrams per liter and 900 milligrams per liter, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.01$?

a. Identify the claim and state H_0 and H_a.

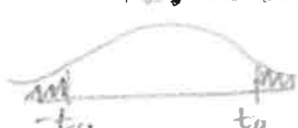
$H_0: \mu = 1890$ (claim)

$H_a: \mu \neq 1890$

b. Find the critical value(s) and identify the rejection region(s). $df = 38$ $\alpha = .01$

two tailed

$t_0 = \pm 2.712$



rej. reg: $t < -2.712$, $t > 2.712$

c. Find the standardized test statistic t.

$\mu = 1890$ $\bar{x} = 2350$ $s = 900$ $n = 39$

$$t = \frac{2350 - 1890}{\frac{900}{\sqrt{39}}} = 3.19$$

which is > 2.712 so it's in the rej. reg.

d. Decide whether to reject or fail to reject the null hypothesis

reject H_0

e. Interpret the decision in the context of the original claim.

There is enough evidence at the 1% level of significance to reject the company's claim that the mean conductivity of the river is 1890 mg/l.

Using P-values with t-tests

Using the table, you can only find two values that the P-value falls between so we will use technology to find the P-value in a t-test.

Press STAT, then arrow to TESTS and select #2: T-Test...

Leave Inpt: on Stats and fill in the rest of the information based on the problem. Then calculate and look to see what it says p is equal to. This is your P-value.

Example 9: A department of motor vehicles office claims that the mean wait time is less than 14 minutes. A random sample of 10 people has a mean wait time of 13 minutes with a standard deviation of 3.5 minutes. At $\alpha = 0.10$, test the office's claim. Assume the population is normally distributed.

$\mu = 14$
$n = 10$
$\bar{x} = 13$
$s = 3.5$
$\alpha = .10$

a. Identify the claim and state H_0 and H_a.

$H_0: \mu \geq 14$

$H_a: \mu < 14$ (claim)

b. Use technology to find the P-value (4 decimal places)

P-value = .1949

c. Decide whether to reject or fail to reject the null hypothesis

.1949 > .10 fail to reject H_0

d. Interpret the decision in the context of the original claim.

There is not enough evidence at the 10% level of significance to support DMV's claim that the mean wait time is less than 14 minutes.

Example 10:

A drug is used to treat leukemia. A random sample of 7 patients using this drug was taken and the remission times in weeks were recorded. Let x be a random variable representing the remission time for all patients using the drug. Assume the distribution is mound shaped and symmetrical. The drug company claims that the mean remission time with this drug treatment is 12.5 weeks. The random sample of 7 patients had a mean of 17.1 weeks with a standard deviation of 10.0 weeks. Do the data indicate that the mean remission time using the drug is different from 12.5 weeks?

Use α =0.01

$\mu = 12.5$ $n = 7$ $\bar{x} = 17.1$ $s = 10$ $\alpha = .01$

a. Identify the claim and state H_0 and H_a.

$H_0: \mu = 12.5$ (claim)

$H_a: \mu \neq 12.5$

b. Use technology to find the P-value

P-value = .2693

c. Decide whether to reject or fail to reject the null hypothesis

.2693 > .01 fail to reject H_0

d. Interpret the decision in the context of the original claim.

There is not enough evidence to reject the drug company's claim that the mean remission time with this drug treatment is 12.5 weeks.